Neutrino Oscillations Induced by Two-loop Radiative Effects

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Abstract

Phenomena of neutrino oscillations are discussed on the basis of two-loop radiative neutrino mechanism. Neutrino mixings are experimentally suggested to be maximal in both atmospheric and solar neutrino oscillations. By using $L_e - L_\mu - L_\tau$ ($\equiv L'$)-conservation, which, however, only ensures the maximal solar neutrino mixing, we find that two-loop radiative mechanism dynamically generates the maximal atmospheric neutrino mixing and that the estimate of $\Delta m^2_\odot/\Delta m^2_{atm} \sim \epsilon m_e/m_\tau$ explains $\Delta m^2_\odot/\Delta m^2_{atm} \ll 1$ because of $m_e/m_\tau \ll 1$, where ϵ measures the breaking of the L'-conservation. Together with $\Delta m^2_{atm} \approx 3 \times 10^{-3}$ eV^2, this estimate yields $\Delta m^2_\odot \sim 10^{-7}$ eV^2 for $\epsilon \sim 0.1$, which corresponds to the LOW solution to the solar neutrino problem. Neutrino mass scale is given by $(16\pi^2)^{-2} m_e m_\tau/M$ ($M \sim 1$ TeV), which is of order 0.01 eV.

1 Introduction

Neutrino oscillations have been long recognized if neutrinos are massive particles [1]. Such oscillations in fact have been recently confirmed by the Super-Kamiokande collaboration [2] and have also been observed for solar neutrinos produced inside the Sun [3]. The recent report from the K2K collaboration [4] has further shown that the atmospheric neutrino oscillations are characterized by $\Delta m^2_{atm} \approx 3 \times 10^{-3}$ eV^2, which implies $\sim 5.5\times 10^{-2}$ eV as neutrino masses. This tiny mass scale for neutrinos can be generated by radiative mechanisms, where the smallness originates from the smallness of radiative effects [7, 8]. Radiative mechanisms uses L=2 interactions given by $\nu_L^{[i}\ell_L^{j]}$ for one-loop radiative effects [7, 9, 10, 11] and by additional $\ell_R^{\{i}\ell_R^{j\}}$ for two-loop radiative effects [8, 12], where i and j denote three families (i,j = 1,2,3).

At the one-loop level, Zee [7] has presented the mechanism that utilizes a new standard Higgs scalar called ϕ' in addition to the standard Higgs scalar, ϕ, both of which are $SU(2)_L$-doublets, and another

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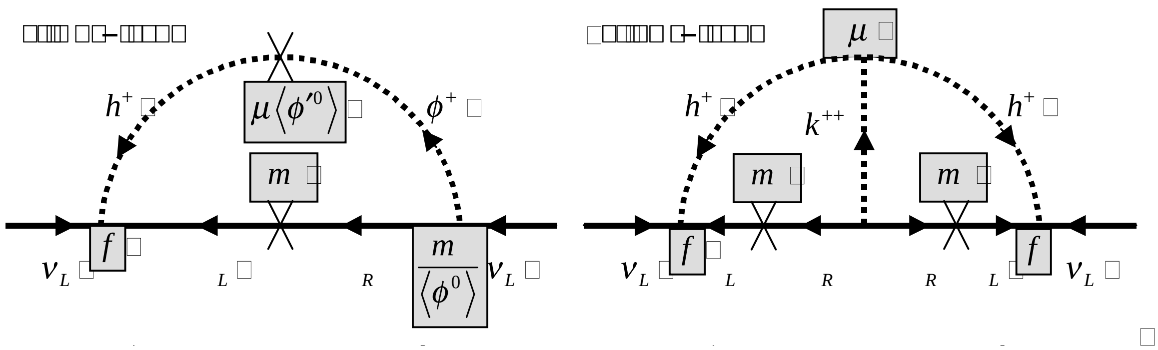

Figure 1: Radiatively generated Majorana neutrino masses: (a) one-loop diagram, (b) two-loop diagram.

singly charged scalar called h^+, which is an $SU(2)_L$-singlet, with the coupling of $f_{[ij]}\nu_L^{[i}\ell_L^{j]}h^+$. The Fermi statistics forces $\nu_L^{[i}\ell_L^{j]}$ to be antisymmetrized with respect to the family indices. After the spontaneous breakdown of $SU(2)_L \times U(1)_Y$, an interaction of $\phi\phi' h^+$ yields the possible mixing of h^+ with ϕ^+ characterized by the scale of μ, which finally induces Majorana neutrino masses. Again, the Fermi statistics forces $\phi\phi'$ to be antisymmetrized with respect to the $SU(2)_L$-indices. Depicted in Figure 1(a) is the diagram for generating Majorana neutrino masses. The order-of-magnitude estimate gives the one-loop neutrino mass, m_ν^{1-loop}, for ν_i-ν_j to be:

$$m_\nu^{1-loop} \sim f_{[ij]}\frac{m_{\ell j}^2}{16\pi^2 M^2}\mu, \tag{1}$$

for $\langle 0|\phi^0|0\rangle \sim \langle 0|\phi'^0|0\rangle$, where M stands for the scale of the model, presumably of order 1 TeV. The factor of $16\pi^2$ in the denominator is specific to one-loop radiative corrections. This estimate turns out to be

$$m_\nu^{1-loop} \sim 2\times 10^3 f_{[i\tau]}\ (\frac{\mu}{100\ \mathrm{GeV}})\ \mathrm{eV}, \tag{2}$$

for $m_{\ell j} = m_\tau$ (j=τ). To obtain $m_\nu \sim 0.1$ eV, we require that

$$f_{[i\tau]} \sim 5\times 10^{-5}, \tag{3}$$

for $\mu \sim 100$ GeV. Therefore, to get tiny neutrino masses of order 0.1 eV, one has to give excessive suppression to the lepton-number violating $\nu\ell$-coupling.

At the two-loop level, additional suppression arises. In addition to h^+, a doubly charged k^{++}-scalar is required to realize the mechanism of the Zee-Babu type [8] and k^{++} couples to a right-handed charged lepton pair via $\ell_R^{\{i}\ell_R^{j\}}k^{++}$ with coupling strength of $f_{\{ij\}}$. Using a possible coupling of this new k^{++} with h^+ via $h^+h^+k^{++\dagger}$, we can find interactions corresponding to Figure 1(b). The order-of-magnitude estimate gives the two-loop neutrino mass, m_ν^{2-loop}, for ν_i-ν_k to be:

$$m_\nu^{2-loop} \sim f_{[ij]}f_{\{jj'\}}f_{[kj']}\frac{m_{\ell j}m_{\ell j'}}{(16\pi^2)^2 M^2}\mu. \tag{4}$$

The factor of $(16\pi^2)^2$ in the denominator is specific to two-loop radiative corrections. This estimate turns out to yield

$$m_\nu^{2-loop} \sim 10 f_{[i\tau]}f_{[\tau\tau]}f_{[j\tau]}\ (\frac{\mu}{100\ \mathrm{GeV}})\ \mathrm{eV}, \tag{5}$$

for $m_{\ell j}, _{\ell j'} = m_\tau$ (j, j'=τ). To obtain $m_\nu \sim 0.1$ eV, thanks to the extra loop-factor of $16\pi^2$, we only require that

$$f_{[i\tau]} \sim 0.1, \tag{6}$$

for $f_{[\tau\tau]} \sim 1$ and $\mu \sim 100$ GeV. Therefore, the two-loop radiative neutrino masses can be of order of 0.1 eV without excessive suppression for relevant couplings [13].

2 Bimaximal Mixing

The observed pattern of neutrino oscillations is consistent with the pattern arising from the requirement of the conservation of the new quantum number $L_e - L_\mu - L_\tau$ ($\equiv L'$) [14]. The $U(1)_{L'}$ symmetry based on the L'-conservation can be used to describe the bimaximal mixing scheme for neutrino oscillations [15, 16]. However, the L'-conservation itself only ensures the maximal solar neutrino mixing but does not determine the atmospheric neutrino mixing angle. In fact, in the one-loop radiative mechanism, fine-tuning of lepton-number violating couplings is necessary to yield bimaximal mixing for atmospheric neutrino oscillations.

In the one-loop radiative mechanism, we have known the form of the neutrino mass matrix, which is given by

$$M_\nu \propto \left.\left(\begin{array}{ccc} 0 & f_{[e\mu]}m_\mu^2 & f_{[e\tau]}m_\tau^2 \\ & 0 & f_{[\mu\tau]}m_\tau^2 \\ & & 0 \end{array} \right)\right|_{sym} \Rightarrow \left(\begin{array}{ccc} 0 & \sim 1 & \sim 1 \\ & 0 & \varepsilon\,(\ll 1) \\ & & 0 \end{array} \right) m, \tag{7}$$

where m stands for the neutrino mass scale. The bimaximal mixing is realized if the couplings satisfy

$$f_{e\mu}m_\mu^2 = f_{[e\tau]}m_\tau^2 \Rightarrow f_{[e\mu]} \gg f_{[e\tau]} \left(\gg f_{[\mu\tau]} \approx 0\right), \tag{8}$$

indicating the fine-tuning of the couplings f's. This fine-tuning is referred to as "inverse hierarchy in the couplings", namely, $f_{[e\mu]} \gg f_{[e\tau]}$ [17]. The L'-conservation gives $f_{[\mu\tau]}$=0. Its tiny breaking effect characterized by the parameter, ε, produces tiny solar neutrino oscillations.

On the other hand, in the two-loop radiative mechanism, we will find the mass matrix [12] given by

$$M_\nu \propto \left.\left(\begin{array}{ccc} 0 & f_{[e\tau]}f_{[e\mu]}m_e m_\tau & f_{[e\tau]}f_{[e\tau]}m_e m_\tau \\ & f_{[e\mu]}f_{[e\mu]}m_e^2 & f_{[e\mu]}f_{[e\tau]}m_e^2 \\ & & f_{[e\tau]}f_{[e\tau]}m_e^2 \end{array} \right)\right|_{sym} \Rightarrow \left(\begin{array}{ccc} 0 & \sim 1 & \sim 1 \\ & \varepsilon & \varepsilon' \\ & & \varepsilon'' \end{array} \right) m. \tag{9}$$

The bimaximal structure is reproduced if

$$f_{[e\tau]}f_{[e\mu]}m_e m_\tau = f_{[e\tau]}f_{[e\tau]}m_e m_\tau \Rightarrow f_{[e\mu]} = f_{[e\tau]}. \tag{10}$$

Therefore, no hierarchy in the couplings is necessary. The breaking of the L'-conservation gives the suppressed entries, ε, ε', ε'', proportional to m_e^2. Therefore, we observe that

$$\Delta m_\odot^2/\Delta m_{atm}^2 \propto m_e/m_\tau, \tag{11}$$

which dynamically guarantees $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ because of $m_\tau \gg m_e$.

In radiative mechanisms, the hierarchy of $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ can also be ascribed to the generic smallness of two-loop radiative effects over one-loop radiative effects [18]. Therefore, we have in hands two dynamical reasons for $\Delta m_{atm}^2 \gg \Delta m_\odot^2$:

$$\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \ll 1 \text{ because} \left\{ \begin{array}{l} 2-\text{loop}/1-\text{loop} \ll 1 \\ m_e/m_\tau \ll 1 \end{array} \right. . \tag{12}$$

Table 1: L and L' quantum numbers.

Fields	$(\nu_{eL}, e_L^-), e_R^-$	$(\nu_{iL}, \ell_{iL}^-), \ell_{iR}^-\vert_{i=\mu,\tau}$	ϕ	h^+	k^{++}	k'^{++}
L	1	1	0	-2	-2	-2
L'	1	-1	0	0	0	-2

3 Two-loop Radiative Neutrino Masses

Interactions that we introduce can be classified by the ordinary lepton number (L) and L'-number of particles, which are listed in the Table 1. The new ingredients that are not contained in the standard model are the $SU(2)_L$-singlet scalars, h^+ and k^{++}. We have further employed an additional k^{++} to be denoted by k'^{++} in order to import the L'- breaking. The L- and L'-quantum number of k'^{++} is also listed in Table 1. Extra L- and L'-conserving Yukawa interactions are given by

$$\left\{ \begin{array}{l} f_{[ej]}\left(\nu_{eL}\ell_L^j - \nu_L^j e_L^-\right) h^+, \\ f_{\{ej\}} e_R^- \ell_R^j k^{++}, \\ \frac{1}{2} f_{\{ee\}} e_R^- e_R^- k'^{++}. \end{array} \right. \tag{13}$$

An L-breaking but L'-conserving interaction is specified by

$$\mu_0 h^+ h^+ k^{++\dagger}, \tag{14}$$

where μ_0 represents a mass scale. An L'-breaking interaction is activated by k'^{++} via

$$\mu_b h^+ h^+ k'^{++\dagger}, \tag{15}$$

where μ_b represents a breaking scale of the L'-conservation.

Yukawa interactions, then, take the form of [1]

$$\begin{aligned} -\mathcal{L}_Y &= \sum_{i=e,\mu,\tau} f_\phi^i \overline{\psi_L^i} \phi \ell_R^i + \sum_{i=\mu,\tau} \left(f_{[ei]} \overline{(\psi_L^e)^c} \psi_L^i h^+ + f_{\{ei\}} \overline{(e_R)^c} \ell_R^i k^{++} \right) \\ &\quad + \frac{1}{2} f_{\{ee\}} \overline{(e_R)^c} e_R k'^{++} + (\text{h.c.}), \end{aligned} \tag{16}$$

and Higgs interactions are described by self-Hermitian terms composed of $\varphi\varphi^\dagger$ ($\varphi = \phi$, h^+, k^{++}, k'^{++}) and by the non-self-Hermitian terms in

$$V_0 = \mu_0 h^+ h^+ k^{++\dagger} + (\text{h.c.}). \tag{17}$$

This coupling softly breaks the L-conservation but preserves the L'-conservation. To account for solar neutrino oscillations, the breaking of the L'-conservation should be included and is assumed to be furnished by

$$V_b = \mu_b h^+ h^+ k'^{++\dagger} + (\text{h.c.}). \tag{18}$$

Neutrino masses are generated by interactions corresponding to the diagrams depicted in Figure 2(a,b). The resulting Majorana neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & \delta_{\mu\mu} & \delta_{\mu\tau} \\ m_{e\tau} & \delta_{\mu\tau} & \delta_{\tau\tau} \end{pmatrix}. \tag{19}$$

[1] The corresponding expression of Eq.(2) in Ref.[19] should read this equation.

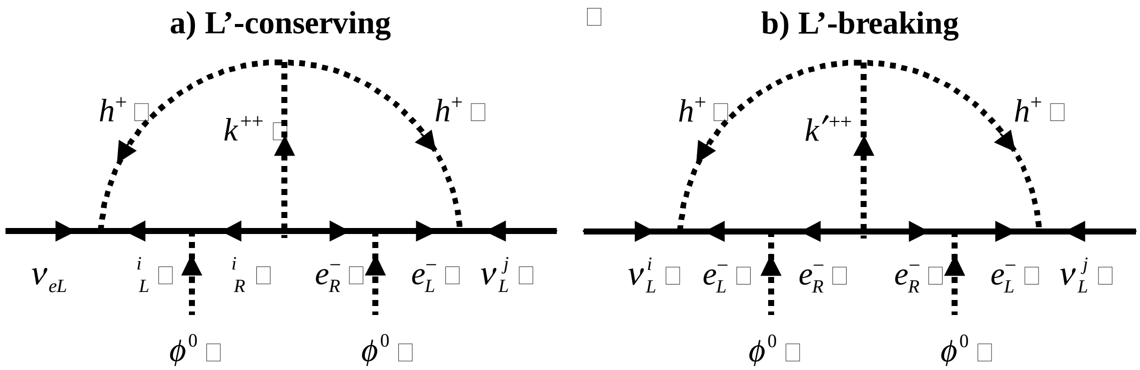

Figure 2: Radiatively generated Majorana neutrino masses: (a) L'-conserving two-loop diagram, (b) L'-breaking two-loop diagram.

Here, the bimaximal structure is controlled by

$$m_{ei} \approx -2f_{[e\tau]}f_{[ei]}f_{\{\tau e\}}\frac{m_\tau m_e}{m_k^2}\mu_0\left[\frac{1}{16\pi^2}\ln\left(\frac{m_k^2}{m_h^2}\right)\right]^2 \ \ (i=\mu,\tau), \tag{20}$$

where the product of m_e and m_τ appears. This is because the exchanged leptons are e and τ as can been seen from Figure 2(a). Tiny splitting is induced by

$$\delta_{ij} \approx -f_{[ei]}f_{[ej]}f_{\{ee\}}\frac{m_e m_e}{m_{k'}^2}\mu_b\left[\frac{1}{16\pi^2}\ln\left(\frac{m_{k'}^2}{m_h^2}\right)\right]^2, \tag{21}$$

where m_e^2 appears because the exchanged leptons are both e and e as can been seen from Figure 2(b). These expressions, Eqs.(20) and (21), are subject to the approximation of $m_{k,k'}^2 \gg$(other mass squared). The detailed derivation of Eqs.(20) and (21) can be found in the Appendix of Ref.[19]. Oscillations are described by these mass parameters:

$$\Delta m_{atm}^2 = m_{e\mu}^2 + m_{e\tau}^2 \left(\equiv m_\nu^2\right), \quad \Delta m_\odot^2 = 4m_\nu\delta m, \tag{22}$$

where

$$\delta m = \frac{1}{2}\left|\delta_{\mu\mu}\cos^2\theta_\nu + 2\delta_{\mu\tau}\cos\theta_\nu\sin\theta_\nu + \delta_{\tau\tau}\sin^2\theta_\nu\right| \tag{23}$$

with

$$\cos\theta_\nu = m_{e\mu}/m_\nu, \quad \sin\theta_\nu = m_{e\tau}/m_\nu. \tag{24}$$

It is thus found that (nearly) bimaximal mixing is reproduced by requiring

$$f_{[e\mu]} \approx f_{[e\tau]}, \tag{25}$$

yielding $\sin 2\theta_\nu \approx 1$. Tiny mass-splitting $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ is ensured by the mass-hierarchy:

$$m_\tau \gg m_e. \tag{26}$$

As a result, we obtain an estimate of the ratio:

$$\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \sim \frac{\mu_b}{\mu_0}\frac{m_e}{m_\tau}\frac{m_k^2}{m_{k'}^2}. \tag{27}$$

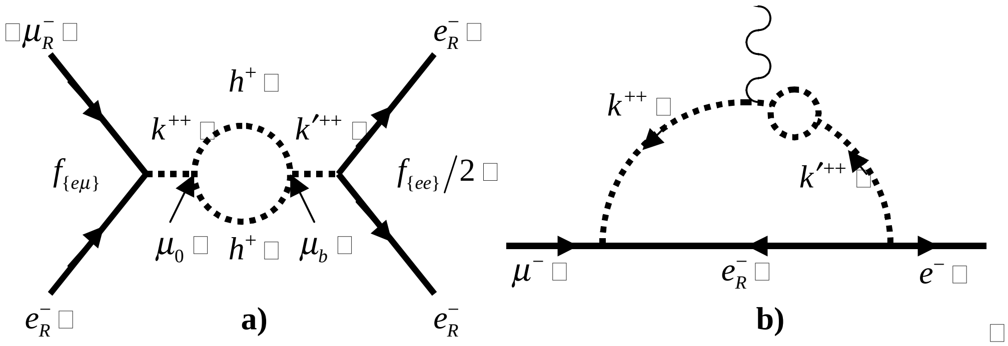

Figure 3: (a) $\mu^- \to e^- e^- e^+$, (b) $\mu^- \to e^- \gamma$.

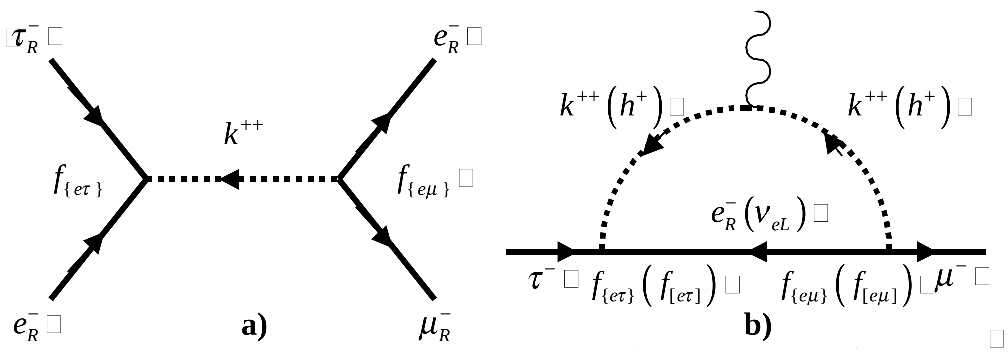

Figure 4: (a) $\tau^- \to \mu^- e^- e^+$, (b) $\tau^- \to \mu^- \gamma$.

From this estimate, we find that

$$\begin{aligned}
&\Delta m^2_\odot \sim 3 \times 10^{-4} \frac{\mu_b}{\mu_0} \Delta m^2_{atm} \ \left(m_k^2 \sim m_{k'}^2\right) \\
&\Rightarrow \Delta m^2_\odot \sim 3 \times 10^{-5} \Delta m^2_{atm} \ (\mu_b \sim \mu_0/10) \\
&\Rightarrow \Delta m^2_\odot \sim 10^{-7}\ \text{eV}^2 \ \left(\Delta m^2_{atm} \sim 3 \times 10^{-3} \text{eV}^2\right). \qquad (28)
\end{aligned}$$

The resulting $\Delta m^2_\odot$ corresponds to the allowed region for the LOW solution to the solar neutrino problem. Since k^{++} and k'^{++} couple to the charged lepton pairs, these scalars produce extra contributions on the well-established low-energy phenomenology. In particular, we should consider effects from $\mu^- \to e^-\gamma$, $e^-e^-e^+$, $e^-e^- \to e^-e^-$ and $\nu_\mu e^- \to \nu_\mu e^-$. The relevant constraints on the parameters associated with the scalars of h^+, k^{++} and k'^{++} are, thus, given by [2]

1. $\mu^- \to e^-e^-e^+$ in Figure 3(a) and $\mu^- \to e^-\gamma$ in Figure 3(b) [20] (forbidden by the L'-conservation), yielding

$$\frac{\xi f_{\{e\mu\}} f_{\{ee\}}}{\bar{m}_k^2} < \left\{ \begin{array}{l} 1.2 \times 10^{-10}\ \text{GeV}^{-2} \text{ from } B\left(\mu^- \to e^-e^-e^+\right) < 10^{-12} \text{ [21]} \\ 2.4 \times 10^{-8}\ \text{GeV}^{-2} \text{ from } B\left(\mu^- \to e^-\gamma\right) < 1.2 \times 10^{-11} \text{ [21]} \end{array} \right., \qquad (29)$$

[2] The constraints of Eqs.(12) and (13) in Ref.[19] should, respectively, be replaced by the corresponding bounds in the items 1, 3 and 4. Namely, $f_{\{11,12\}}$ should read $f_{\{11,12\}}/2$ in Ref.[19].

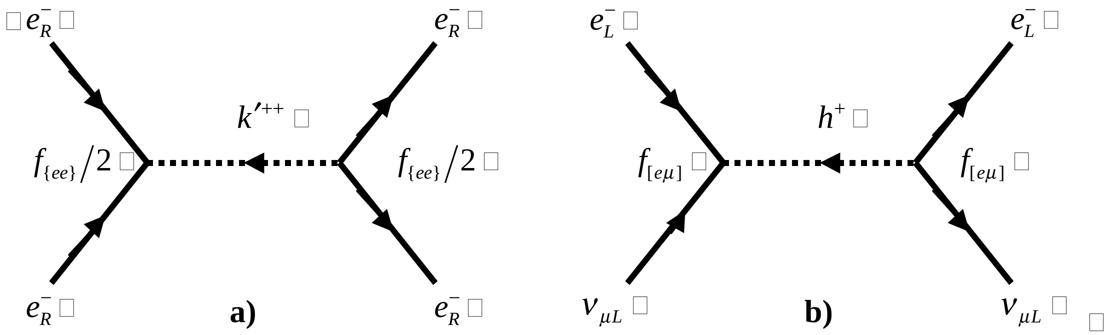

Figure 5: (a) $e^-e^- \to e^-e^-$, (b) $\nu_\mu e^- \to \nu_\mu e^-$.

where $\bar{m}_k \sim m_k \sim m_{k'}$ and ξ estimated to be

$$\xi \sim \frac{1}{16\pi^2}\frac{\mu_b\mu_0}{\bar{m}_k^2} \ \ (\ll 1) \tag{30}$$

reads the suppression due to the approximate L'-conservation,

2. $\tau^- \to \mu^-e^-e^+$ in Figure 4(a) and $\tau^- \to \mu^-\gamma$ in Figure 4(b) (allowed by the L'-conservation), yielding

$$\left|\frac{f_{\{e\tau\}}f_{\{e\mu\}}}{\bar{m}_k^2}\right| < \left\{ \begin{array}{l} 2.1\times10^{-7}\ \mathrm{GeV}^{-2}\ \mathrm{from}\ B\left(\tau^-\to\mu^-e^-e^+\right) < 1.7\times10^{-6}\ [21] \\ 4.2\times10^{-6}\ \mathrm{GeV}^{-2}\ \mathrm{from}\ B\left(\tau^-\to\mu^-\gamma\right) < 1.1\times10^{-6}\ [21] \end{array}\right. ,$$
$$\left|\frac{f_{[e\tau]}f_{[e\mu]}}{m_h^2}\right| < 4.2\times10^{-6}\ \mathrm{GeV}^{-2}\ \mathrm{from}\ B\left(\tau^-\to\mu^-\gamma\right), \tag{31}$$

3. $e^-e^- \to e^-e^-$ [22] in Figure 5(a), yielding

$$\left|\frac{f_{\{ee\}}}{m_{k'}}\right|^2 < 4.8\times10^{-5}\ \mathrm{GeV}^{-2}, \tag{32}$$

4. $\nu_\mu e^- \to \nu_\mu e^-$ [23] in Figure 5(b), yielding

$$\left|\frac{f_{[e\mu]}}{m_h}\right|^2 < 1.7\times10^{-6}\ \mathrm{GeV}^{-2}. \tag{33}$$

It should be noted that the leading contribution of h^+ to $\mu^- \to e^-\gamma$, which gives the most stringent constraint on h^+, is forbidden by the $U(1)_{L'}$-invariant coupling structure.

Typical parameter values are so chosen to satisfy these constraints:

$$\left.\begin{array}{l} f_{[e\mu]} = f_{[e\tau]} \approx 2e \\ f_{\{ee\}} = f_{\{e\tau\}} \approx e \end{array}\right\}\ \text{to suppress higher} - \text{order effects},$$
$$\left.\begin{array}{l} m_h \approx 350\ \mathrm{GeV} \\ m_k = m_{k'} \approx 2\ \mathrm{TeV} \\ \mu_0 \approx 1.5\ \mathrm{TeV} \\ \mu_\mathrm{b} \approx \mu_0/10 \end{array}\right\}\ \ \text{to suppress exotic contributions.} \tag{34}$$

We obtain the following numerical values:

$$\left\{ \begin{array}{l} \Delta m^2_{atm} \approx 2.4 \times 10^{-3}\ \mathrm{eV}^2, \\ \Delta m^2_\odot \approx 10^{-7}\ \mathrm{eV}^2. \end{array} \right. \tag{35}$$

Therefore, we in fact successfully explain phenomena of atmospheric and solar neutrino oscillations characterized by $\Delta m^2_{atm} \approx 2.4 \times 10^{-3}\mathrm{eV}^2$ and $\Delta m^2_\odot \approx 10^{-7}\mathrm{eV}^2$.

4 Summary

We have discussed how neutrino oscillations arise from two loop-radiative mechanism, which exhibits

1. bimaximal mixing due to the $L_e - L_\mu - L_\tau$ conservation via the coupling of $e^-\tau^-k^{++}$,
2. dynamically induced tiny mass-splitting for solar neutrino oscillations due to the smallness of m_e via $e^-e^-k^{\prime ++}$.

The interactions required to generate two-loop Majorana neutrino masses are specified by

$$\left\{ \begin{array}{l} f_{[ei]}\left(\nu_{eL}\ell^i_L - \nu^i_L e^-_L\right) h^+ \\ f_{\{ei\}} e^-_R \ell^j_R k^{++} \\ \frac{1}{2} f_{\{ee\}} e^-_R e^-_R k^{\prime ++} \end{array} \right. (i=\mu,\tau) \oplus \left\{ \begin{array}{l} \mu_0 h^+ h^+ k^{++\dagger} \\ \mu_b h^+ h^+ k^{\prime ++\dagger} \end{array} \right. . \tag{36}$$

The resulting mass scale for neutrino masses is determined by

$$\frac{m_\tau m_e}{(16\pi^2)^2 m^2_k}\mu_0 \sim \frac{m_\tau m_e}{(16\pi^2)^2 m_k} \sim 10^{-2}\ \mathrm{eV}. \tag{37}$$

Thus, to obtain the neutrino mass of order of 0.01 eV is a natural consequence without fine-tuning of coupling parameters. And the hierarchy of $\Delta m^2_{atm} \gg \Delta m^2_\odot$ is expressed by the estimate

$$\Delta m^2_\odot \sim \frac{\mu_b}{\mu_0}\frac{m_e}{m_\tau}\frac{m^2_k}{m^2_{k'}}\Delta m^2_{atm}, \tag{38}$$

which ensures $\Delta m^2_{atm} \gg \Delta m^2_\odot$ because of $m_\tau \gg m_e$. [3] This estimation yields the LOW solution to the solar neutrino problem.

It should be finally noted that

- since the L'-conservation forbids primary flavor-changing processes involving e^-, the coupling strengths of h^+ and k^{++} to leptons are not severely constrained and can be as large as $\mathcal{O}(e)$,
- characteristic signatures of h^+ include

$$B(h^+ \to e^+ \not{E}_T) \approx 2B(h^+ \to \mu^+ \not{E}_T) \approx 2B(h^+ \to \tau^+ \not{E}_T) \tag{39}$$

since $f_{[e\mu]} \approx f_{[e\tau]}$, which should be compared with [24]

$$B(h^+ \to e^+ \not{E}_T) \approx B(h^+ \to \mu^+ \not{E}_T) \gg B(h^+ \to \tau^+ \not{E}_T) \tag{40}$$

in the one-loop radiative mechanism with $f_{[e\mu]} \gg f_{[e\tau]} \gg f_{[\mu\tau]}$ [17].

Acknowledgments

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[3] One should be aware of higher-order contributions found by Lavoura in Ref.[12]. The (1,1)-entry of M_ν, which vanishes up to the two-loop level, is induced by the four-loop diagram shown in Figure 6. The contributions are at most characterized by $\delta \sim (16\pi^2)^{-1}\xi m^2_\tau/m^2_k$, which should be compared with $m^2_e/m^2_{k'}$. Our parameter-setting in Eq.(34) gives $\delta \sim 2m^2_e/m^2_k$, which turns out to be $\mathcal{O}(m^2_e/m^2_{k'})$. Therefore, our estimate of Eq.(38) remains valid to predict $\Delta m^2_\odot$ from Δm^2_{atm}.

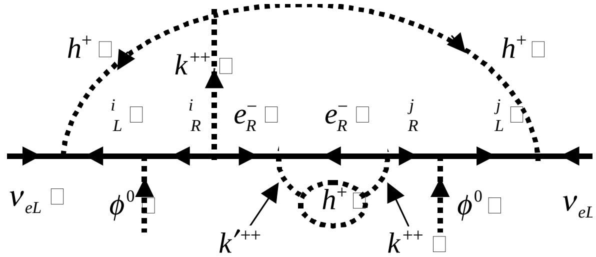

Figure 6: Four loop-diagram for ν_e-ν_e.

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